Gravitational Collapse and Expansion of Charged Anisotropic Cylindrical Source

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Abstract

In this paper, we have discussed the gravitational collapse and expansion of charged anisotropic cylindrically symmetric gravitating source. To this end, the generating solutions of Einstein-Maxwell field equations for the given source and geometry have been evaluated. We found the auxiliary solution of the filed equations, this solution involves a single function which generates two kinds of anisotropic solutions. Every solution can be expressed in terms of arbitrary function of time that has been chosen arbitrarily to fit the various astrophysical time profiles. The existing solutions predict gravitational collapse and expansion depending on the choice of initial data. Instead of base to base collapse, in the present case, wall to wall collapse of the cylindrical source has been investigated. We have found that the electromagnetic field is responsible for the enhancement of anisotropy in collapsing system.

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1 Introduction

The study of gravitational collapse started from the pioneer work of Chandrasekhar (1936). Later on, Oppenheimer and Snyder (19390investigated that the spherically symmetric homogenous dust collapse leads to the formation of black hole (BH). Initially, it was argued that the homogeneity and

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spherical symmetry of the collapsing model are responsible for the formation of BH. However, it was found (Lemaitre 1933) that inhomogeneous dust collapse would end as shell crossing and shell focusing singularities. After many years, it was proved (Joshi 1997) that the shell crossing singularity is naked, while shell focusing singularity might represent BH, depending on the choice of initial data of collapse. Hence, it was concluded that the homogeneity of the collapsing model is not a sufficient condition for the formation of BH. In order to generalize the collapsing matter, it becomes necessary to study the collapse of more realistic matter with non-vanishing pressure.

Misner and Sharp (1964) studied perfect fluid collapse and found BH as the end state of gravitational collapse. Also, Herrera and Santos (1997) explored the properties of anisotropic self-gravitating spheres and discussed their stability using the perturbation method. Herrera and his collaborators (Herrera et al.2008a, Herrera et al. 2008b, Herrera at al. 1989, Herrera et al. 2009, Herrera et al. 2010, Herrera et al. 2012) have discussed the stability and applications of anisotropic solutions to stellar collapse. Recently, Glass (2013) has formulated a generating solution of anisotropic spherically symmetric solutions which reveal either expansion and collapse depending on the choice of time profile of the solutions. This work has been extended for plane symmetric anisotropic source and charged anisotropic sphere by Abbas (2014a, 2014b). The present paper is the cylindrical version of the these papers.

The current observational evidences of gravitational waves (through the detectors LIGO (Abramovici et al. 1992) and GEO (L$\ddot{u}$ck and GEO 600 Team 1997) have increased the interest to study the gravitational collapse in cylindrically symmetric systems. The spherical systems are simple and do not provide non-trivial examples in the generic gravitational collapse. Therefore, the study of cylindrical collapse is much important as compared to spherical systems. Some numerical studies (Piran 1978) provide the generation of gravitational waves from cylindrical collapse. These results have been extended analytically (Nakao and Morisawa 2004) to study gravitational waves during cylindrical gravitational collapse. Sharif and Ahmad (2007) generalized this work for two perfect fluid cylindrical collapse and discussed the generation of gravitational waves. Di Prisco et al.(2009) studied the shearfree gravitational collapse of the anisotropic fluid in the cylindrically symmetric spacetime.

The applications of the electromagnetic field in astronomy and astrophysics is an active research domain. A lot of work has been devoted to discuss the collective effects of electromagnetic and gravitational fields. Till

now, there is a little progress about the effects of electromagnetic field on gravitational collapse of stars. Thorne (1965) studied cylindrically symmetric gravitational collapse with magnetic field and concluded that magnetic field can prevent the collapse of cylinder before singularity formation. Ardvan and Partovi (1977) investigated dust solution of the field equations with electromagnetic field and found that the electrostatic force is balanced by gravitational force during collapse of charged dust. The effects of electromagnetic field on structure scalars and dynamics of self-gravitating objects have been explored by Herrera and his collaborators (Herrera et al. 2011, Diprisco et al. 2007). Sharif and his collaborators (Sharif and Bhatti 2012a, Sharif and Bhatti 2012b, Sharif and Bhatti 2013a, Sharif and Bhatti 2013b, Sharif and Bhatti 2014, Sharif and Yousaf 2012, Sharif and Kausar 2011) have extended this work for cylindrical and plane symmetries in GR as well as in $f(R)$ gravity with electromagnetic field.

This paper is organized as follows: In section **2**, charged anisotropic cylindrical source and Einstein-Maxwell equations have presented. Section **3** is devoted to the generating solutions which represent gravitational collapse and expansion of the self-gravitating charged cylinder. We summaries the results of the paper in the last section.

2 Matter Distribution and Field Equations

This section deals with the interior matter distribution and corresponding Einstein-Maxwell's equations. The non-static spacetime with cylindrical symmetry in the interior region of a star is given by (Sharif and Bhatti 2015)

$$ds^2 = -A^2dt^2 + B^2dr^2 + C^2d\theta^2 + dz^2, \tag{1}$$

where $-\infty<t<\infty,\ 0 \leq r < \infty,\ 0 \leq \theta \leq 2\pi,\ -\infty < z < \infty$ are the restrictions on the coordinates of cylinder. In the interior region of cylindrically symmetric star, we have considered the charged anisotropic fluid for which energy-momentum is given by (Di Prisco et al. 2009)

$$\begin{aligned} T_{\alpha\beta} &= (\mu + p_r)v_\alpha v_\beta - (p_r - p_z)s_\alpha s_\beta - (p_r - p_\theta)k_\alpha k_\beta \\ &+ p_r g_{\alpha\beta} + \frac{1}{4\pi}\left(F^\gamma_\alpha F_{\beta\gamma} - \frac{1}{4}F^{\gamma\delta}F_{\gamma\delta}g_{\alpha\beta}\right), \end{aligned} \tag{2}$$

where $\mu,\ p_r,\ p_\theta$,and p_z are the energy density, pressures in $r,\ \theta$ and z directions, respectively. Further, v_α is four-velocity and $s_\alpha,\ k_\alpha$ are four-vectors.

Also, $F_{\alpha\beta}=-\phi_{\alpha,\beta}+\phi_{\beta,\alpha}$ is the Maxwell field tensor with four-potential ϕ_α. Moreover, s_α and k_α are the unit four-vectors which satisfy the following relations

$$s^\alpha s_\alpha = k^\alpha k_\alpha = 1, \quad v^\alpha v_\alpha = -1, \quad s^\alpha k_\alpha = v^\alpha k_\alpha = v^\alpha s_\alpha = 0.$$

In comoving coordinate system, these quantities can be written as

$$k_\alpha = C\delta^2_\alpha, \quad v_\alpha = -A\delta^0_\alpha, \quad s_\alpha = \delta^3_\alpha. \tag{3}$$

The Maxwell field's equations are

$$F^{\alpha\beta}_{\ \ ;\beta} = 4\pi J^\alpha, \quad F_{[\alpha\beta;\gamma]} = 0,$$

where J_α is the four-current. In comoving coordinates, the charge inside the cylinder is at rest, so we can define the four-potential and four-current as follows:

$$\phi_\alpha = \phi\delta^0_\alpha, \quad J^\alpha = \zeta v^\alpha,$$

where $\zeta(r,t)$ and $\phi(r,t)$ are charge density and scalar potential, respectively. The expansion scalar is

$$\Theta = \frac{1}{A}\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right). \tag{4}$$

We define the dimensionless anisotropy as follows:

$$\Delta a = \frac{p_r - p_\theta}{p_r}. \tag{5}$$

The corresponding Einstein-Maxwell's equations have the following form

$$\kappa\left(\mu - \frac{\pi}{2}E^2\right)A^2 = \frac{\dot{B}\dot{C}}{BC} + \left(\frac{A}{B}\right)^2\left(\frac{B'C'}{BC} - \frac{C''}{C}\right), \tag{6}$$

$$0 = \frac{\dot{C}'}{C} - \frac{\dot{B}C'}{BC} - \frac{\dot{C}A'}{CA}, \tag{7}$$

$$\kappa\left(p_r+\frac{\pi}{2}E^2\right)B^2 = \frac{A'C'}{AC}+\left(\frac{B}{A}\right)^2\left(-\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}\right), \qquad (8)$$

$$\kappa\left(p_\theta-\frac{\pi}{2}E^2\right) = \left(\frac{1}{AB}\right)\left(\frac{\dot{A}\dot{B}}{A^2}-\frac{A'B'}{B^2}-\frac{\ddot{B}}{A}+\frac{A''}{B}\right), \qquad (9)$$

$$\begin{aligned}\kappa\left(p_z-\frac{\pi}{2}E^2\right) &= -\frac{\ddot{B}}{A^2B}+\frac{A''}{AB^2}-\frac{\ddot{C}}{A^2C}-\frac{A'B'}{AB^3}+\frac{\dot{A}}{A^3}\left(\frac{\dot{C}}{C}+\frac{\dot{B}}{B}\right)\\ &- \frac{C'}{B^2C}\left(\frac{B'}{B}+\frac{A'}{A}\right)-\frac{\dot{B}\dot{C}}{A^2BC}+\frac{C''}{B^2C}, \qquad (10)\end{aligned}$$

where $E=\frac{s}{2\pi C}$ with $s(r)=4\pi\int_0^r \zeta BCdr$ is the total amount of charge per unit length of the cylinder.

Thorne (1965) defined the mass function for cylindrical geometry in the form of gravitational C-energy per unit length of the cylinder. The specific energy $m=\tilde{E}l$ (l is the length of cylinder, i.e., g_{zz}) of cylindrical geometry (1) in the presence of electric charge is given by (Sharif and Bhatti 2015)

$$m(t,r)=sC+\frac{1}{8}\left[1-\left(\frac{C'}{B}\right)^2+\left(\frac{\dot{C}}{A}\right)^2\right]. \qquad (11)$$

The auxiliary solution of Eq.(7) is

$$A=\frac{\dot{C}}{C^\alpha}, \quad B=C^\alpha, \qquad (12)$$

where α is arbitrary constant. Now using Eq.(12) in Eq.(4), we get the following form of expansion scalar

$$\Theta=(1+\alpha)C^{\alpha-1}. \qquad (13)$$

For $\alpha>-1$ and $\alpha<-1$, we obtain expanding and collapsing solutions respectively. Using Eq.(12), in Eqs.(6)-(10), we get the following form of

Einstein-Maxwell's Equations:

$$\begin{aligned}
8\pi\left(\mu-\frac{s^2}{8\pi C^2}\right) &= \alpha C^{2(\alpha-1)}+C^{-2\alpha}\left(\frac{\alpha C'C''}{C^2}-\frac{C''}{C}\right), &(14)\\
8\pi\left(p_r+\frac{s^2}{8\pi C^2}\right) &= \alpha C^{2(2\alpha-1)}+\frac{C'\dot{C}'}{\dot{C}C^{(2\alpha+1)}}-\frac{\alpha C'}{C^{(2\alpha+2)}}, &(15)\\
8\pi\left(p_\theta+\frac{s^2}{8\pi C^2}\right) &= \alpha C^{2(\alpha-1)}\left(\frac{C\ddot{C}}{\dot{C}^2}-\alpha\right)-\left(\frac{\dot{C}'C-\alpha\dot{C}C'}{C^{2(\alpha+1)}}\right)C'\\
&- \alpha C^{\alpha}\left((\alpha-1)(\frac{\dot{C}}{C})^2+\frac{\ddot{C}C^{\alpha}}{C\dot{C}}\right)+\left(\frac{(1-\alpha)(C''\dot{C}+\dot{C}'C')}{C^{2\alpha+1}}\right)\\
&- \left(\frac{(\alpha+1)C'(C\dot{C}'-\alpha\dot{C}C')}{C^{(\alpha+2)}}\right), &(16)
\end{aligned}$$

$$\begin{aligned}
8\pi\left(p_z+\frac{s^2}{8\pi C^2}\right) &= \left(\frac{\alpha(\alpha-1)}{C^2}+\frac{\alpha\ddot{C}}{C\dot{C}^2}\right)C^{2\alpha}+\frac{(1-\alpha)(C''\dot{C}+C'\dot{C}')}{\dot{C}C^{3\alpha+1}}\\
&- \frac{(\alpha+1)\left(CC'\dot{C}'-\alpha\dot{C}C'^2\right)}{C^{2(\alpha+1)}\dot{C}}-\frac{\ddot{C}C^{2\alpha-1}}{\dot{C}^2}-\frac{\alpha\dot{C}'C'C}{C^{2\alpha+1}\dot{C}}\\
&+ \frac{\alpha^2C'^2}{C^{2(\alpha+1)}}+\frac{(\alpha+1)C^{2(2\alpha-1)}\left(C\ddot{C}-\alpha\dot{C}^2\right)}{\dot{C}^2}+\frac{C''-\alpha C^{3(\alpha-1)}}{C^{2\alpha+1}}\\
&- \frac{CC'\dot{C}'}{\dot{C}C^{2(\alpha+1)}}. &(17)
\end{aligned}$$

For specific values of $C(r,t)$ and α, we can find anisotropic configuration. In this case mass function along with electromagnetic field given in Eq.(11) takes the following form:

$$8m-8sC-1=C^{2\alpha}-\frac{C'^2}{C^{2\alpha}} \qquad (18)$$

If $C'=C^{2\alpha}$ then above equation gives

$$C=\frac{1}{s}\left(m-\frac{1}{8}\right) \qquad (19)$$

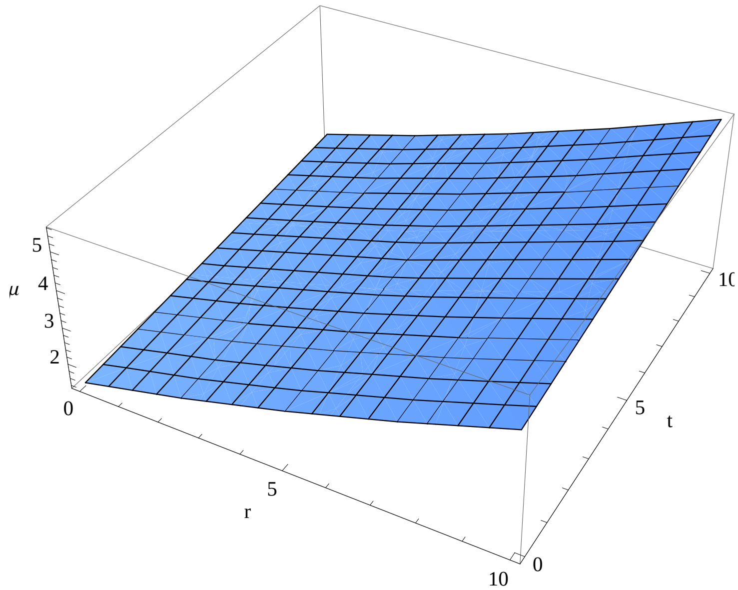

Figure 1: Density variation for $s=2$ and $h_1(t)=1+t$.

where $m>\frac{1}{8}$. This implies that gravitational collapse leads to the formation of a trapping surface at $C=\frac{1}{s}\left(m-\frac{1}{8}\right)$. Also, the integration of trapping condition $C'=C^{2\alpha}$ yields

$$C^{(1-2\alpha)}=(1-2\alpha)r+h(t), \tag{20}$$

where $h(t)$ is an arbitrary function.

3 Generating solution

For negative and positive values of α , we have collapsing and expanding solutions, respectively as follows:

3.1 Gravitational Collapse with $\alpha=-\frac{3}{2}$

For collapse, expansion scalar will be negative, from Eq.(11),$\Theta<0$ when $\alpha>-1$. We assume that $\alpha=-\frac{3}{2}$ and the condition $C'=C^{2\alpha}$, leads to $C'=C^{-}3$, the integration of this equation yields,

$$C_{trap}=(4r+h_1(t))^{\frac{1}{4}}, \tag{21}$$

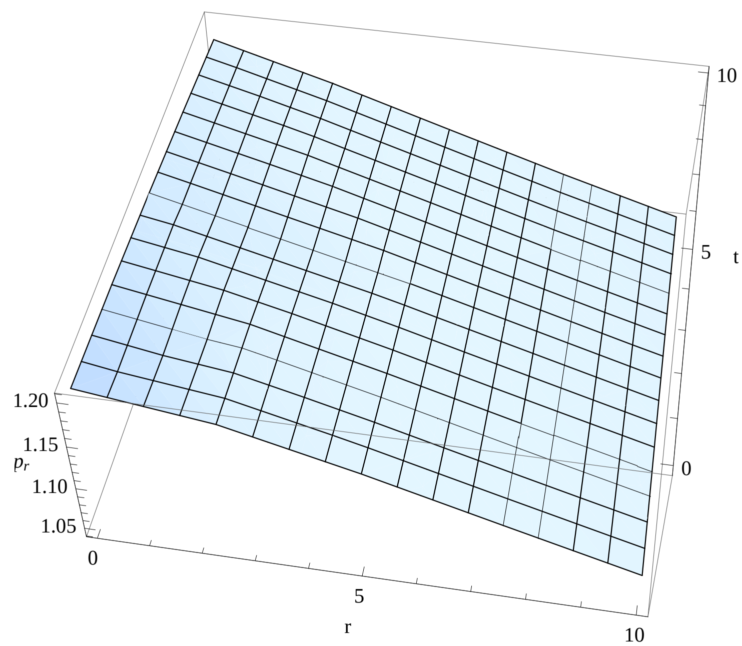

Figure 2: Radial pressure variation for $s = 2$ and $h_1(t) = 1 + t$.

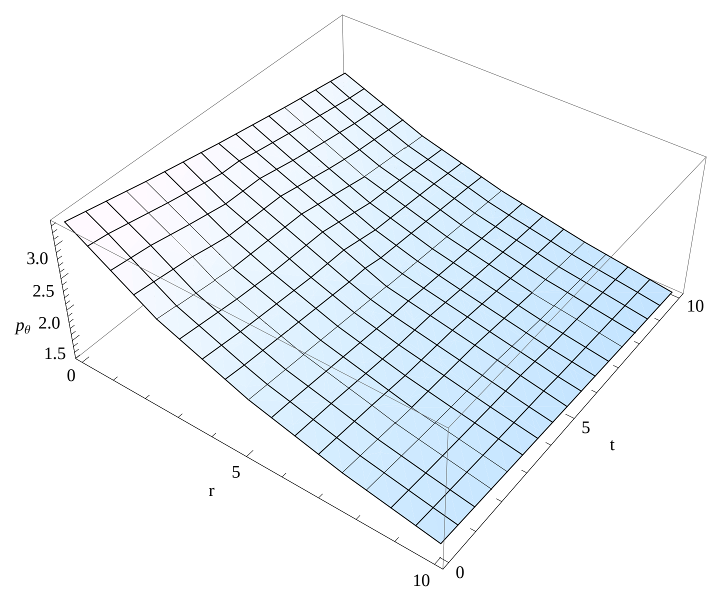

Figure 3: Transverse pressure variation for $s = 2$ and $h_1(t) = 1 + t$.

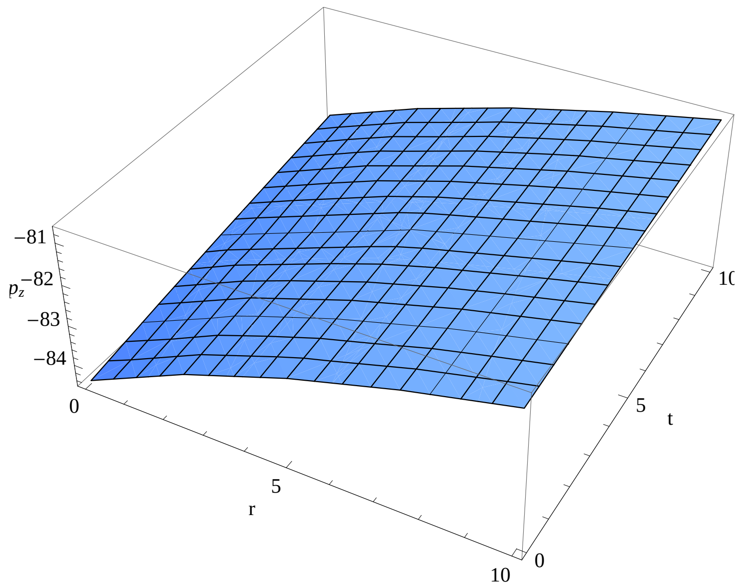

Figure 4: Longitudinal pressure variation for $s = 2$ and $h_1(t) = 1 + t$.

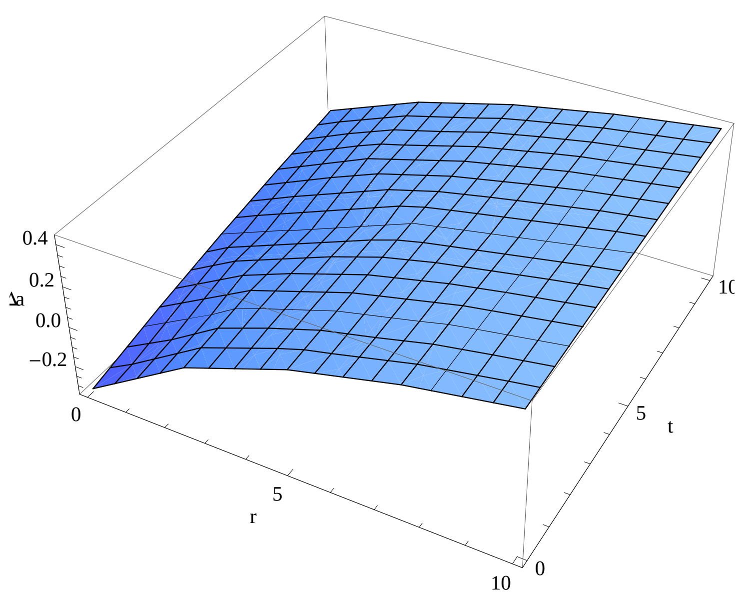

Figure 5: Dimensionless anisotropic parameter variation for $s = 2$ and $h_1(t) = 1 + t$.

where $h_1(t)$ is arbitrary function of time. For $\alpha = -\frac{3}{2}$, Eqs.(14)-(17) are given by

$$8\pi\mu = -C^3\left(\frac{3C'C''+CC'''}{2C^2}\right)-\frac{3C^{-3}-s^2}{2} \quad (22)$$

$$8\pi p_r = \frac{C\left(3C'\dot{C}+CC'\dot{C}'\right)}{2\dot{C}}-\frac{3C^{-5}-s^2}{2C^2} \quad (23)$$

$$\begin{aligned} 8\pi p_\theta &= \frac{C\left(5CC''\dot{C}+3CC'\dot{C}'-3C'^2\dot{C}\right)}{2}+\frac{2C^4C'\dot{C}'+3C^3C'^2\dot{C}}{4C^{\frac{7}{2}}} \\ &- \frac{15\dot{c}^2}{4C^{\frac{7}{2}}}+\frac{6C\dot{C}\ddot{C}-6\ddot{C}C+9\dot{C}^2}{4C^3\dot{C}^2}-\frac{s^2}{C^2} \quad (24) \end{aligned}$$

$$\begin{aligned} 8\pi p_z &= \left(\frac{2C^5\dot{C}^2+3C^3C'\dot{C}+6C^3C'\dot{C}'\dot{C}+9C^3C'^2\dot{C}+6C^3\dot{C}^2+2C^4C'\dot{C}'}{4C^2\dot{C}}\right) \\ &+ \frac{6C^4\dot{C}{+}4C^4C''\dot{C}+4s^2\dot{C}}{4C^2\dot{C}}+C^{\frac{1}{5}}\left(\frac{12\ddot{C}-15\dot{C}^2}{4\dot{C}^2}\right)-\frac{\ddot{C}}{C^4\dot{C}^2}-\frac{C\ddot{C}+6C^2}{4C^8\dot{C}^2} \\ &+ C^{-\frac{7}{2}}\left(\frac{3}{2}+\frac{5}{2}(\frac{C''}{C}+\frac{C'\dot{C}'^2}{C\dot{C}})\right). \quad (25) \end{aligned}$$

Using Eq.(22) in the above equations, we get the following form of field equations:

$$8\pi\mu = -\frac{3}{2(4r+h_1)^{5/4}}+\frac{s^2}{\sqrt{4r+h_1}}+\left(\frac{9+6(4r+h_1)}{2(4r+h_1)^{\frac{9}{4}}}\right), \quad (26)$$

$$8\pi P_r = -\frac{s^2}{\sqrt{4r+h_1}}+\frac{3}{2}\left(-\frac{1}{(4r+h_1)^2}-\frac{2}{(4r+h_1)^{5/4}}+\frac{1}{\sqrt{4r+h_1}}\right) \quad (27)$$

$$\begin{aligned} 8\pi p_\theta &= \left(\frac{3\left(144(4r+h_1)^{9/8}\dot{h_1}^2-4\left(1+24(4r+h_1)^{3/8}\right)\dot{h_1}^3\right.}{64(4r+h_1)^{19/8}\dot{h_1}^2}\right) \\ &- \left(\frac{5h_1'^4-128(4r+h_1)^{17/8}\ddot{h_1}+32(4r+h_1)^{11/8}\dot{h_1}\ddot{h_1}\Big)}{64(4r+h_1)^{19/8}\dot{h_1}^2}\right), \quad (28) \end{aligned}$$

$$
\begin{aligned}
8\pi p_z &= \frac{4s^2}{\sqrt{4r+h_1}} + \frac{3}{4}\left(-\frac{20}{(4r+h_1)^{23/8}} + \frac{1}{(4r+h_1)^2} - \frac{5}{(4r+h_1)^{5/4}}\right) \\
&+ \frac{3}{2(4r+h_1)^{7/8}} - \frac{12}{(4r+h_1)^{1/5}} - 5(4r+h_1)^{1/20} \\
&+ \frac{1}{8}\left(1 - \frac{3}{(4r+h_1^2} + \frac{3}{\sqrt{4r+h}}\right)\dot{h_1} + \frac{15\dot{h_1}^2}{64(4r+h_1)^{5/4}} \\
&+ \frac{2\left(-\frac{1}{4r+h_1} - \frac{2}{(4r+h_1)^{1/4}} + 6(4r+h_1)^{4/5}\right)\dot{h_1}}{\dot{h_1}^2}. \qquad (29)
\end{aligned}
$$

The dimensionless measure of anisotropy is given by the following equation:

$$
\begin{aligned}
\Delta \mathrm{a} &= \left[\frac{1\left(-\frac{6}{(4r+h_1^2} - \frac{39}{(4r+h_1^{5/4}} + \frac{6}{\sqrt{4r+h_1}} - \frac{4s^2}{\sqrt{4r+h_1}}\right)}{4\left(-\frac{s^2}{\sqrt{4r+h_1}} + \frac{3}{2}\left(-\frac{1}{(4r+h_1)^2} - \frac{2}{(4r+h_1)^{5/4}} + \frac{1}{\sqrt{4r+h_1}}\right)\right)}\right] \\
&+ \left[\frac{\left[\left(12+96(4r+h_1)^{3/8}\right)\dot{h_1}^3 + 5\dot{h_1}^4\right]}{64\left(-\frac{s^2}{\sqrt{4r+h_1}} + \frac{3}{2}\left(-\frac{1}{(4r+h_1)^2} - \frac{2}{(4r+h_1)^{5/4}} + \frac{1}{\sqrt{4r+h_1}}\right)\right)}\right] \\
&+ \left[\frac{128(4r+h_1)^{17/8}\dot{h_1} - 32(4r+h_1)^{11/8}\dot{h_1}\ddot{h_1}}{64\left(-\frac{s^2}{\sqrt{4r+h_1}} + \frac{3}{2}\left(-\frac{1}{(4r+h_1)^2} - \frac{2}{(4r+h_1)^{5/4}} + \frac{1}{\sqrt{4r+h_1}}\right)\right)}\right]. \qquad (30)
\end{aligned}
$$

For $\alpha = \frac{-5}{2}$, we obtain $\Theta < 0$ and matter density increases for the arbitrary choice charge s and time profile $h_1 = 1 + t$. As density is increasing (see figure.**1**), so cylinder goes on collapsing to a point. In this case, the length of cylinder is constant, i.e., $g_{zz} = 1$, therefore base to base collapse is impossible, there is only possibility of wall to wall collapse. The anisotropic parameter Δa changes its sign from negative to positive. The anisotropy will be directed outward when $p_\theta > p_r$, this implies that $\Delta a > 0$ and directed inward when $p_\theta < p_r$ implying $\Delta a < 0$. In this case $\Delta a > 0$, for larger value of r as shown in figures **5**. This implies that anisotropic force allows the construction of more massive star while $\Delta a < 0$ near the center, so there exist an attractive force. When we talk about an external (electromagnetic) field in gravitational, then there is an external force which may distort the generic properties of spacetime effectively. So, in the present case electromagnetic field enhances the anisotropy and the homogeneity of collapsing star.

3.2 Expansion with $\alpha = \frac{3}{2}$

We know that for expansion, the expansion scalar will be positive, from Eq.(11), $\Theta < 0$, when $\alpha > 0$. In this case assume that $C = (r^2+r_0^2)^{-1}+h_2(t)$, where $h_2(t)$ and r_0 are arbitrary function and constant respectively. For $\alpha = \frac{3}{2}$, Eqs.(14)-(17) take the following form:

$$
\begin{aligned}
8\pi\mu &= \frac{s^2}{C^2} + \frac{3C}{2} + C^{-3}\left(\frac{3C'C''}{2C^2} - \frac{C''}{C}\right), &(31)\\
8\pi p_r &= -\frac{s^2}{C^2} + \frac{3C^4}{2} + \frac{C'\dot{C}'}{C^4\dot{C}} - \frac{3C'}{2C^5}, &(32)\\
8\pi p_\theta &= -\frac{s^2}{C^2} + \frac{3C'}{2}\left(\frac{C\ddot{C}}{\dot{C}^2} - \frac{3}{2}\right) - \left(\frac{\dot{C}C' - \frac{3}{2}\dot{C}C'}{C^5}\right)C' - \frac{3}{2}\left(\frac{\dot{C}^3 + 2C\ddot{C}}{2C^{\frac{3}{2}}}\right)\\
&+ \frac{\left(C''C' + \dot{C}'C'\right) - 5C^{\frac{1}{2}}C'\left(C\dot{C}' - \frac{3}{2}C'\dot{C}\right)}{2C^4}, &(33)\\
8\pi p_z &= \frac{s^2}{C^2} - \frac{3}{2}\left(\frac{1}{2C} + \frac{\ddot{C}}{\dot{C}^2}\right)C^2 - \frac{\left(C''\dot{C} + C'\dot{C}'\right) - 5C'C''\left(CC'' - \frac{3}{2}C'\dot{C}\right)}{2C^{\frac{11}{2}}\dot{C}}\\
&- \frac{C^2\ddot{C}}{\dot{C}^2} - \left(\frac{6CC'\dot{C}' - 9\dot{C}C'^2}{4C^5\dot{C}}\right) + \frac{C^5\ddot{C}}{2\dot{C}^2} - \frac{15C^4}{4} - C^{-5}\left(\frac{3C''^2 + 2C\dot{C}' - 3C'\dot{C}}{2}\right)\\
&- \frac{3C^{\frac{-1}{2}}}{2} + \frac{C''}{C^4}. &(34)
\end{aligned}
$$

If $F(t,r)=1+h_2(t)(r^2+r_0^2)$ and $C=\frac{F}{r^2+r_0^2}$ then Eqs.(32)-(34) become:

$$
\begin{aligned}
8\pi\mu &= \left(\frac{s^2(F-1)^2}{h_2(t)F^2}\right)+\frac{3h_2(t)F}{8\pi(F-1)}-\frac{2(r_0^2-3r^2)(F^2-F-3r)}{8\pi h_2(t)F^5} &(35)\\
8\pi p_r &= \left(-\left(\frac{s(F-1)}{h_2(t)F}\right)^2\right)+\frac{3}{16\pi}\left(\frac{Fh_2(t)}{F-1}\right)^4+\frac{3r}{8\pi F^5}\left(\frac{F-1}{h_2(t)}\right)^3 &(36)\\
8\pi p_\theta &= \frac{1}{2}\left(\left(-\frac{2s(r^2+r_0^2)}{F}\right)^2+\frac{2r(6r^2-2r_0^2+15r(r^2+r_0^2)\sqrt{F})\dot{h_2(t)}}{(r^2+r_0^2)^{\frac{3}{2}}F^4}\right)\\
&+ \frac{4r^2(r^2+r_0^2)\dot{h_2(t)}}{F^5}+\frac{3}{2}\left((\frac{F}{(r^2+r_0^2)})(-\frac{3}{2}+\frac{F\ddot{h_2}}{\dot{h_2}^2(r^2+r_0^2)})\right)\\
&- \frac{3}{2}\left(\left(\frac{F}{(r^2+r_0^2)}\right)^{\frac{3}{2}}\right)\left(\frac{\dot{h_2(t)}^2(r^2+r_0^2)^2}{2F^2}+\frac{\sqrt{F}\ddot{h_2}}{\sqrt{(r^2+r_0^2)}\dot{h_2}}\right) &(37)
\end{aligned}
$$

$$
\begin{aligned}
8\pi p_z &= \frac{1}{2}\left[\left(\frac{s(r^2+r_0^2)}{F}\right)^2-\frac{3\sqrt{(r^2+r_0^2)}}{\sqrt{F}}+\frac{18r^2(r^2+r_0^2)}{F^5}-\frac{4(r^2+r_0^2)(r_0^2-3r^2)}{F^4}\right]\\
&- \frac{6r(r^2+r_0^2)(2r+(r^2+r_0^2)^2)\dot{h_2}}{F^5}+\frac{5F^4(-\frac{3}{2}\dot{h_2}^2+(r^2+r_0^2)F\ddot{h_2}}{(r^2+r_0^2)^5(\dot{h_2})^2}\\
&- \frac{2(-3r^2+r_0^2)\left(20r+20r(-3r^4+r_0^4-2r^2r_0^2)h_2+(r^2+r_0^2)^2(r^8+r_0^8)\dot{h_2}\right)}{(r^2+r_0^2)^{\frac{7}{2}}F^{\frac{11}{2}}\dot{h_2}}\\
&+ \frac{(r^2+r_0^2)^2(4r^6r_0^6+6r^4r_0^4+4r_0^2r^6-30r^2)\dot{h_2}}{(r^2+r_0^2)^{\frac{7}{2}}F^{\frac{11}{2}}\dot{h_2}}-\frac{2F^2\ddot{h_2}}{(r^2+r_0^2)^2\dot{h_2}}\\
&- \frac{3F}{2(r^2+r_0^2)^2}\left(1+\frac{2F\ddot{h_2}}{(r^2+r_0^2)^2\dot{h_2}}\right). &(38)
\end{aligned}
$$

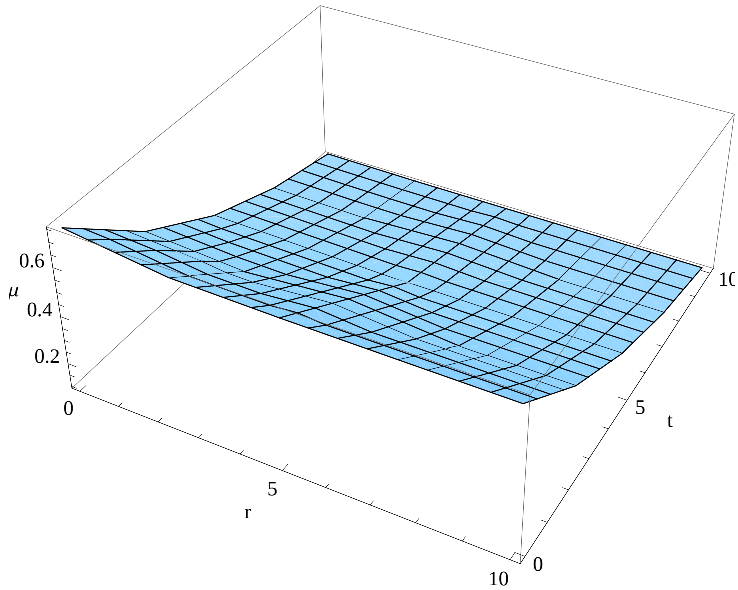

Figure 6: Density variation for $s = 2$ and $h_2(t) = 1 + t$.

The dimensionless measure of anisotropy in this case takes the following form:

$$\begin{aligned}\Delta a &= \left[\frac{3F^4}{(r^2+r_0^2)^4} + \frac{6r(r^2+r_0^2)^3}{F^5} - \frac{4r^2(r^2+r_0^2)\dot{h}_2}{F^5} + \left(\frac{3F^{\frac{3}{2}}}{(r^2+r_0^2)^{\frac{3}{2}}}\right)\right] \\ &- \left[\frac{2r(-2r_0^2+6r^2+15r\sqrt{(r^2+r_0^2)F}\dot{h}_2)}{(r^2+r_0^2)F^4} - \left(\frac{3F}{(r^2+r_0^2)}\right)\left(\frac{-3}{2}+\frac{F\ddot{h}_2}{(r^2+r_0^2)h\dot{(t)}^2}\right)\right] \\ &\times \left[\frac{(r^2+r_0^2)h\dot{(t)}^2}{2F^2} + \frac{\sqrt{F}\ddot{h}_2}{\sqrt{(r^2+r_0^2)}\dot{h}_2}\right]\left[\frac{1}{-\frac{s^2((r^2+r_0^2)^2)}{F^2}+\frac{3}{2}\left(\frac{F^4}{(r^2+r_0^2)^4}\right)+\frac{3r(r^2+r_0^2)^3}{F^5}}\right] \qquad (39)\end{aligned}$$

For $\alpha = \frac{3}{2}$, we have $\Theta > 0$ and matter density decreases (see figure **6**) for the arbitrary choice of charge and time profile $h_2 = 1 + t$. In this case p_r and p_θ are positive and negative (see figure **7,8**), respectively. This implies that there exist anisotropy due to opposite behaviour of pressure components. From figure **10** $\Delta a > 0$, there exists repulsive force which causes the expansion of matter in this case. The expansion process causes to separate the charges apart from each other, this results to weak electromagnetic field intensity.

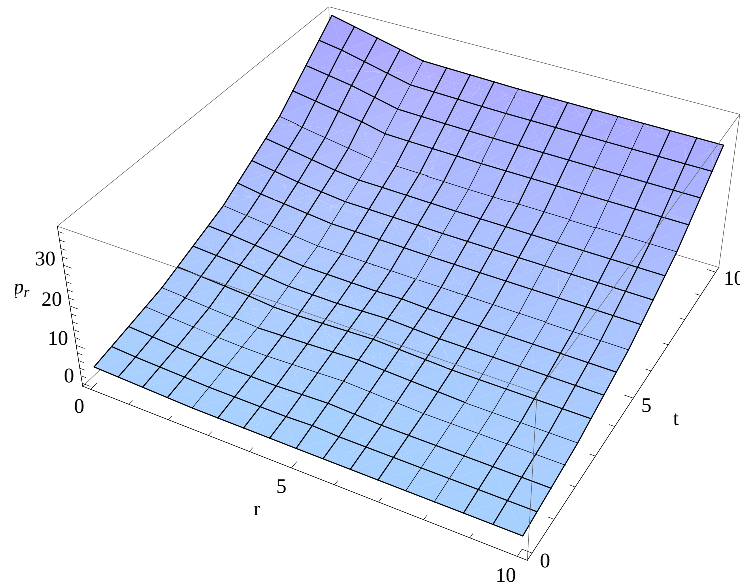

Figure 7: Radial pressure variation for $s = 2$ and $h_2(t) = 1 + t$.

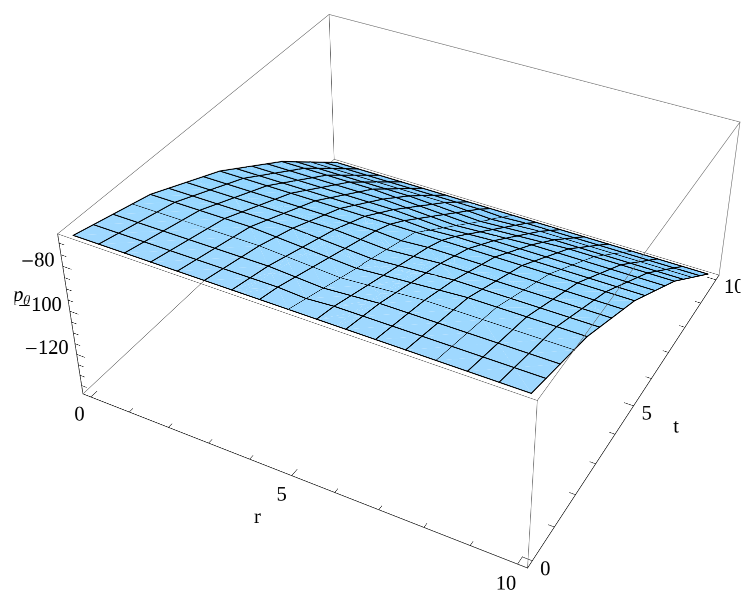

Figure 8: Transverse pressure variation for $s = 2$ and $h_2(t) = 1 + t$.

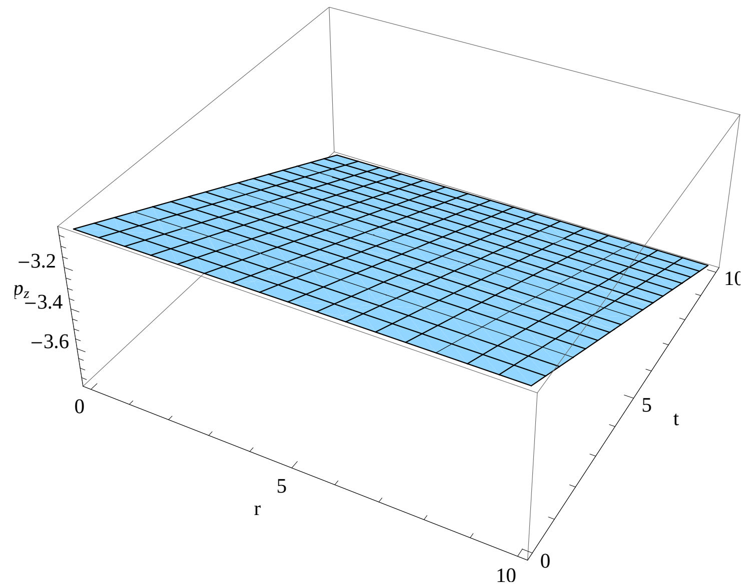

Figure 9: Longitudinal pressure variation for $s = 2$ and $h_2(t) = 1 + t$.

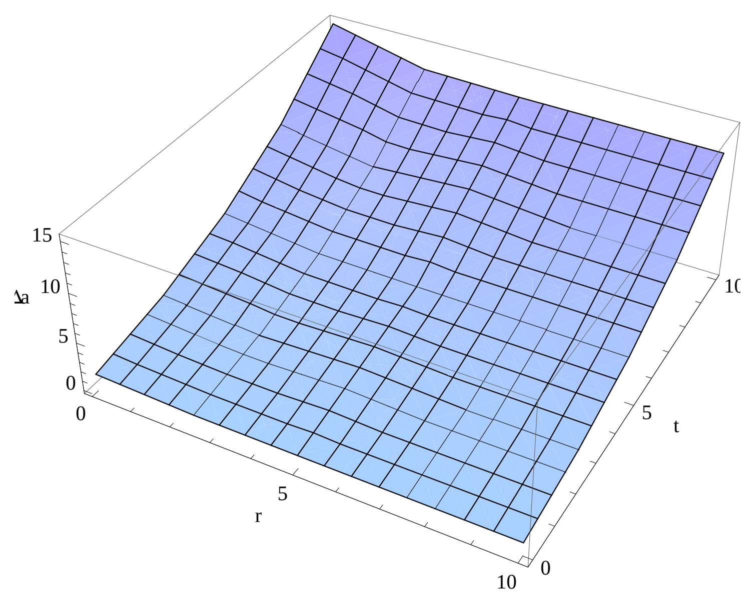

Figure 10: Dimensionless anisotropic parameter variation for $s = 2$ and $h_2(t) = 1 + t$.

4 Concluding Remarks

During the last few decades, there has been a growing interest to study the relativistic anisotropic systems due to the existence of such systems in astronomical objects. The exact solutions of anisotropic sources are helpful to determine the anisotropy of the universe during any era. The effects of anisotropy on the late-time expansion of inhomogeneous universe have been studied by Barrow and Maartens (1998). They remarked that the decrease in shear anisotropy can be figure out by measuring the anisotropic pressure of the cosmological model. Herrera and Santos (1997) pointed out that the phase transition in anisotropic highly dense system would occur during the gravitational collapse. Further, they concluded that such system may transited to a pion condensed phase, where a soften equation of state can provide enough exhausted energy.

This paper is aimed to study the generating solution of Einstein-Maxwell field equations with anisotropic cylindrically symmetric fluid. We have used the auxiliary solution of one field equation to determine the solution of the remaining equations. The application of assumed solution in expansion scalar, allows us to determine the range of free constant α, for which expansion scalar Θ is positive or negative, leading to expansion and collapse. The C-energy analogous to Misner-Sharp mass has been calculated with the contribution. We have impose the condition $C' = C^{2\alpha}$, on the mass function which leads to the existence of trapping horizon at $C = \frac{1}{s}\left(m - \frac{1}{8}\right)$, provided $m > \frac{1}{8}$. In this case curvature singularity is hidden at the center of trapping horizon.

The expansion scalar $\Theta = (2+\alpha)C^{(1-\alpha)}$, becomes $\Theta = 0$, for $\alpha = -1$, $\Theta > 0$, for $\alpha > -1$, $\Theta < 0$, for $\alpha < -1$, which corresponds to bouncing, expansion and collapse, respectively. In other words $\Theta > 0$, $\alpha \in (0, \infty)$, and $\Theta < 0$ for $\alpha \in (-\infty, 0)$ which corresponds to expansion and collapse, respectively. For the sake of simplicity, we have taken $\alpha = -\frac{3}{2}$ for gravitational collapse and $\alpha = \frac{3}{2}$ for expansion, explicitly. The full dynamics of the system has been discussed in both cases. The matter density is increasing/dcreasing function with arbitrary choice of charge parameter and time profiles. The pressures p_r, p_θ and p_z are different in both cases, therefore the pressure anisotropy is non-vanishing in both cases. This anisotropy is increasing function in both cases, it is due to presence of electromagnetic field, as it produces an external repulsive force to distort the geometry of the star.

We would like to mention this work with **charged plane symmetric source** is in progress.

5 Conflict of Interest

The authors declare that they have no conflict of interest.

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